

C. U. SHAH UNIVERSITY

Summer Examination-2022

Subject Name : Metric Space

Subject Code: 4SC05MES1

Branch: B.Sc. (Mathematics)

Semester: 5

Date: 26/04/2022

Time: 11:00 To 02:00

Marks: 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

-
- Q-1 Attempt the following questions:** [14]
- a) Let (X, d) be a metric space and $E \subset X$. Then set E is said to be dense set if ... (01)
- 1) $E' = X$
 - 2) $E' = E$
 - 3) $\bar{E} = X$
 - 4) $\bar{E} = E$
- b) Which of the following subset of \mathbf{R} is not closed? (01)
- 1) $\{1,2, \dots, 10\}$
 - 2) $\{1,2,3, \dots\}$
 - 3) $[0,100]$
 - 4) $(-1,6]$
- c) If $E = [1,3]$ is subset of metric space \mathbf{R} then $E^\circ =$ ____ (01)
- 1) $(1,3)$
 - 2) $[1,3)$
 - 3) $(1,3)$
 - 4) $[1,3]$
- d) Define : Compact Set (01)
- e) Define : Interior Point (01)
- f) Check whether the statement is true or false: If $A \subseteq B$ then $A^\circ \subseteq B^\circ$. (01)
- g) Define : Metric Space (01)
- h) Check whether the statement is true or false: Every closed and bounded subset of the real line is not compact. (01)
- i) Find A° for $A = (0,1]$ (01)
- j) Check whether the statement is true or false: Let A be connected subset of metric space X and B be a subset of X such that $A \subseteq B \subseteq \bar{A}$ then B is also connected. (01)
- k) Let $X = \mathbf{R}$ and $A = \emptyset$ then find $\text{int } A$ and $\text{ext } A$. (02)
- l) Define : Continuous function in Metric space (02)



Attempt any four questions from Q-2 to Q-8

- Q-2 Attempt all questions** [14]
- a) Prove: i) Finite intersection of open sets of metric space is an open set. (06)
ii) Arbitrary intersection of closed sets of metric space is a closed set.
- b) Let (X, d) be a metric space and $E \subset X$. If a' is a limit point of E then show that there are infinitely many points of E in every neighborhood of a' . (04)
- c) Define : Closed Set .Show that every finite subset of metric space is closed. (04)
- Q-3 Attempt all questions** [14]
- a) Let $E_n = (c - \frac{1}{n}, c + \frac{1}{n})$ where $c \in \mathbf{N}$ is constant and $n \in \mathbf{N}$. Compute $\bigcup_{n=1}^{\infty} E_n$ and $\bigcap_{n=1}^{\infty} E_n$ and determine whether they are open or closed ? (06)
- b) Let $X = \mathbf{R}$ and define $d: \mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$ by $d(x, y) = |x - y|$, then prove that (X, d) is metric space. (05)
- c) Define (i) Derived Set (ii) Dense Set (03)
- Q-4 Attempt all questions** [14]
- a) Let (X, d) be a metric space and $d_1: X \times X \rightarrow \mathbf{R}$ defined by $d_1(x, y) = \frac{d(x, y)}{1 + d(x, y)}$ then prove that d_1 is also a metric on X . (06)
- b) Show that distinct points of metric space have different neighborhoods. (05)
- c) If (X, d) is a metric space and $A, B \subset X$ with $A \subset B$, then show that $\bar{A} \subset \bar{B}$. (03)
- Q-5 Attempt all questions** [14]
- a) For a non-empty subset A of metric space (X, d) show that the function $f: X \rightarrow \mathbf{R}$ defined by $f(x) = d(x, A)$, $x \in X$ is uniformly continuous. Also show that $f(x) = 0$ if and only if $x \in \bar{A}$. (07)
- b) Let (X, d) be a complete metric space and $\{F_n\}$ be a decreasing sequence of non-empty closed subsets of X such that $d(F_n) \rightarrow 0$ as $n \rightarrow \infty$, then show that $F = \bigcap_{n=1}^{\infty} F_n$ contains exactly one point. (07)
- Q-6 Attempt all questions** [14]
- a) Prove that the derived set of any subset of metric space is a closed set. (07)
- b) Let (X, d_1) and (Y, d_2) be any two metric space, then prove that $f: X \rightarrow Y$ is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y . (07)
- Q-7 Attempt all questions** [14]
- a) State and prove Banach Fixed Point Theorem. (07)
- b) Let (X, d) be a metric space .If $\{x_n\}$ is convergent sequence of points of X then show that $\{x_n\}$ is Cauchy sequence. (04)
- c) Show that the sets $A = (5,6)$ and $B = (6,8)$ are separated sets of metric space \mathbf{R} . (03)
- Q-8 Attempt all questions** [14]
- a) Define :Cantor Set.Show that Cantor set is a closed set. (07)
- b) Show that every compact subset A of metric space (X, d) is bounded. (05)
- c) Give an example of subsets A and B of metric space \mathbf{R} such that $(A \cap B)' \neq A' \cap B'$. (02)

